

# 1-loop Corrections to the $\rho$ Parameter in the Left-Right Twin Higgs Model

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**Abstract.** We implement a one-loop analysis of the  $\rho$  parameter in the Left Right Twin Higgs model, including the logarithmically enhanced contributions from both fermion and scalar loops. Numerical results show that the one-loop contributions are dominant over the tree level corrections in most regions of parameter space. The experimentally allowed values of  $\rho$ -parameter divide the allowed parameter space into two regions; less than 670 GeV and larger than 1100 GeV roughly, for symmetry breaking scale  $f$ . Our numerical analysis significantly reduces the parameter space which are favorably accessible to the LHC.

**PACS.** 12.60.-i Models beyond the standard model – 12.15.Lk Electroweak radiative corrections

## 1 Introduction

The Standard Model (SM) has excellently described high energy physics up to energies of  $\mathcal{O}(100)$  GeV. The only undetected constituent of the SM, up to now, is a Higgs boson which is required to explain the generation of fermion and gauge boson masses. Theoretically, the Higgs boson mass squared is quadratically sensitive to any new physics scale beyond the Standard Model (BSM) which may arise at higher energy scales and hence stabilization of the Higgs mass squared prefers the energy scale at which the BSM turns up to be lowered to  $\mathcal{O}(1)$  TeV. On the other hand, electroweak precision measurements with naive naturalness assumption raise the energy scale of the BSM up to 100 TeV or even higher. Hence, there remains a tension between theory and experiment associated with the stabilization of the SM Higgs mass. But with the start-up of the LHC the tension may be relaxed by direct observation of the BSM at TeV energy scale. The idea of little Higgs originates in the speculation that the SM Higgs may be a pseudo-Nambu-Goldstone boson [1, 2, 3, 4, 5, 6, 7]. Stabilization of the Higgs mass in the little Higgs theories is achieved by the "collective symmetry breaking" which naturally renders the SM Higgs mass much smaller than the symmetry breaking scale. The distinct elements of little Higgs models are a vector-like heavy top quark and various scalar and vector bosons. The former is universal while the latter is model-dependent. Both of them contribute significantly to one-loop processes and hence establish strict constraints on the parameter space of little Higgs models. At worst, electroweak precision tests push up the symmetry breaking scale

to 5 TeV or higher, and regenerate significant fine-tuning in the Higgs potential. Twin Higgs idea shares the same origin with that of little Higgs in that the SM Higgs is a pseudo-Nambu-Goldstone boson [8]. But rather than using collective symmetry breaking to stabilize the Higgs mass squared it makes use of additional discrete symmetry. In other words, the discrete symmetry ensures the absence of quadratic divergence in the Higgs mass squared. The twin Higgs mechanism is realized by identifying the discrete symmetry with left-right symmetry in the left-right model [9]. The left-right twin Higgs (LRTH) model contains  $U(4)_1 \times U(4)_2$  global symmetry as well as  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  gauge symmetry. The left-right symmetry acts on only the two  $SU(2)$ 's gauge symmetry. A pair of vector-like heavy top quarks play a key role at triggering electroweak symmetry breaking just as that of the little Higgs theories. Besides, the other Higgs particles acquire large masses not only at quantum level but also at tree level. These heavy Higgs bosons make the model deliver rich phenomenology at the LHC [10]. But theoretically, they lead to large radiative corrections to one-loop processes and, in return, the allowed parameter space can be reduced significantly. In this paper, we perform a one-loop analysis of the  $\rho$ -parameter in the LRTH model to reduce the parameter space. This is based on the original work with Jae Yong Lee, KIAS [11].

## 2 Left-Right Twin Higgs Model [10]

The LRTH model is based on the global  $U(4)_1 \times U(4)_2$  symmetry, with a locally gauged subgroup  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ . A pair of Higgs fields,  $H$  and  $\hat{H}$ , are introduced and each transforms as  $(4, 1)$  and  $(1, 4)$

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respectively under the global symmetry. They are written as

$$H = \begin{pmatrix} H_L \\ H_R \end{pmatrix}, \quad \hat{H} = \begin{pmatrix} \hat{H}_L \\ \hat{H}_R \end{pmatrix}, \quad (1)$$

where  $H_{L,R}$  and  $\hat{H}_{L,R}$  are two component objects which are charged under the  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  as

$$H_L \text{ and } \hat{H}_L : (2, 1, 1), \quad H_R \text{ and } \hat{H}_R : (1, 2, 1). \quad (2)$$

The global  $U(4)_1(U(4)_2)$  symmetry is spontaneously broken down to its subgroup  $U(3)_1(U(3)_2)$  with VEVs

$$\langle H \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix}, \quad \langle \hat{H} \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \hat{f} \end{pmatrix}. \quad (3)$$

Each spontaneous symmetry breaking results in seven Nambu-Goldstone bosons, which are parameterized as

$$H = f e^{\pi/f} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \pi = \begin{pmatrix} -\frac{N}{2\sqrt{3}} & 0 & 0 & h_1 \\ 0 & -\frac{N}{2\sqrt{3}} & 0 & h_2 \\ 0 & 0 & -\frac{N}{2\sqrt{3}} & C \\ h_1^* & h_2^* & C^* & \frac{\sqrt{3}N}{2} \end{pmatrix}, \quad (4)$$

where  $\pi$  is the corresponding Goldstone field matrix.  $N$  is a neutral real field  $C$  and  $C^*$  are a pair of charged complex scalar fields, and  $h_{SM} = (h_1, h_2)$  is the SM  $SU(2)_L$  Higgs doublet.  $\hat{H}$  is parameterized in the identical way by its own Goldstone boson matrix,  $\hat{\pi}$ , which contains  $\hat{N}$ ,  $\hat{C}$ , and  $\hat{h} = (\hat{h}_1^+, \hat{h}_2^0)$ . In turn, two  $U(4)/U(3)$  symmetry breaking left with fourteen Nambu-Goldstone bosons. The linear combination of  $C$  and  $\hat{C}$ , and the linear combination of  $N$  and  $\hat{N}$  are eaten by the gauge bosons of  $SU(2)_R \times U(1)_{B-L}$ , which is broken down to the  $U(1)_Y$ . The orthogonal linear combinations, a charged complex scalar  $\phi^\pm$  and a neutral real pseudoscalar  $\phi^0$ , remain as Nambu-Goldstone bosons. On top of that, the SM Higgs acquires a VEV,  $\langle h_{SM} \rangle = (0, v/\sqrt{2})$ , and thereby electroweak symmetry  $SU(2)_L \times U(1)_Y$  is broken down to  $U(1)_{EM}$ . But  $\hat{h}$ 's do not get a VEV and remain as Nambu Goldstone bosons. These Nambu Goldstone bosons acquire masses through quantum effects and/or soft symmetry breaking terms, so called  $\mu$ -terms,

$$V_\mu = -\mu_r^2 (H_R^\dagger \hat{H}_R + c.c.) + \hat{\mu}^2 \hat{H}_L^\dagger \hat{H}_L, \quad (5)$$

which contribute to the Higgs masses at tree level. Because of the extend gauge symmetry, there are extra gauge bosons besides the SM gauge bosons,  $W_H$  and  $Z_H$ , masses of which are proportional to  $f$  and  $\hat{f}$ . The existence of the extra gauge bosons would be the typical feature of the generic left-right symmetric models.

To cancel the quadratic sensitivity of the Higgs mass to the top quark loops, a pair of vector-like, charge 2/3 fermion ( $Q_L, Q_R$ ) are incorporated into the top Yukawa sector,

$$\mathcal{L}_{Yuk} = y_L \bar{Q}_{L3} \tau_2 H_L^* Q_R + y_R \bar{Q}_{R3} \tau_2 H_R^* Q_L - M \bar{Q}_L Q_R + h.c., \quad (6)$$

where  $\tau_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ,  $Q_{L3} = -i(u_{L3}, d_{L3})$  and  $Q_{R3} = (u_{R3}, d_{R3})$  are the third generation up- and down-type quarks, respectively. The left-right parity indicates  $y_L = y_R (\equiv y)$ . The mass parameter  $M$  is essential to the top mixing. The value of  $M$  is constrained by the  $Z \rightarrow b\bar{b}$  branching ratio. It can be also constrained by the oblique parameters, which we will do in the letter. Furthermore, it yields large log divergence of the SM Higgs mass. To compensate for it the heavy gauge bosons also get large masses by increasing the value of  $\hat{f}$ . Therefore it is natural for us to take  $M \lesssim yf$ .

### 3 Results and Discusion

The  $Z$ -pole,  $W$ -mass, and neutral current data can be used to search for and set limits on deviations from the SM. In the article we concentrate particularly on the the  $\rho$ -parameter, which is defined as

$$\rho \equiv \frac{M_W^2}{M_Z^2 c_\theta^2}. \quad (7)$$

The effective leptonic mixing angle  $s_\theta^2 (= 1 - c_\theta^2)$  at the  $Z$ -resonance is defined as the ratio of the electron vector to axial vector coupling constants to the  $Z$ -boson,

$$\frac{Re(g_V^e)}{Re(g_A^e)} \equiv 4s_\theta^2 - 1, \quad (8)$$

where the coupling constants of a fermion  $\psi$  to the gauge boson  $X$  is given as,

$$\mathcal{L} = i\bar{\psi}_1 \gamma_\mu (g_V + g_A \gamma_5) \psi_2 X^\mu. \quad (9)$$

Using the procedure in Ref. [12], we can calculate the 1-loop corrected  $W$  boson mass

$$M_W^2 = \frac{1}{2} \left[ a(1 + \Delta\hat{r}) + \sqrt{a^2(1 + \Delta\hat{r})^2 + 4a\Pi^{WW}(0)} \right], \quad (10)$$

with  $a \equiv \frac{\pi\alpha(M_Z)}{\sqrt{2}G_F s_\theta^2}$ , and the definition of  $\Delta\hat{r}$  is

$$\Delta\hat{r} = -\frac{\Delta s_\theta^2}{s_\theta^2} - \frac{Re(\Pi^{ZZ}(M_Z^2))}{M_Z^2} + \Pi^{\gamma\gamma'}(0) + 2\left(\frac{g_V^e - g_A^e}{Q_e}\right) \frac{\Pi^{\gamma Z}(0)}{M_Z^2} - \frac{c_\theta^2 - s_\theta^2}{c_\theta s_\theta} \frac{Re(\Pi^{\gamma Z}(M_Z^2))}{M_Z^2}. \quad (11)$$

The 1-loop corrected  $\rho$  parameter is then obtained using Eq. (7) with the  $M_W^2$  value predicted in Eq. (10). For doing the calculation concerning the precision measurements, the standard experimental values are necessary which play as input parameters. Here, we use the following experimentally measured values for the input parameters [13, 14]:

$$G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}, \quad (12)$$

$$M_Z = 91.1876(21) \text{ GeV}, \quad (13)$$

$$\alpha(M_Z)^{-1} = 127.918(18), \quad (14)$$

$$s_\theta^2 = 0.23153(16). \quad (15)$$

We also take the top and bottom quark masses as [13, 15]

$$m_t = 172.3 \text{ GeV}, \quad m_b = 3 \text{ GeV}, \quad (16)$$

where  $m_t$  is the central value of the electroweak fit and  $m_b$  is the running mass at the  $M_Z$  scale with  $\overline{MS}$  scheme. Including all the SM corrections (top quark loop, bosonic loops), we take the allowed range of  $\rho$  parameter as [13]

$$1.00989 \leq \rho^{exp} \leq 1.01026. \quad (17)$$

The input parameters of the LRTH model [9] are as follows:

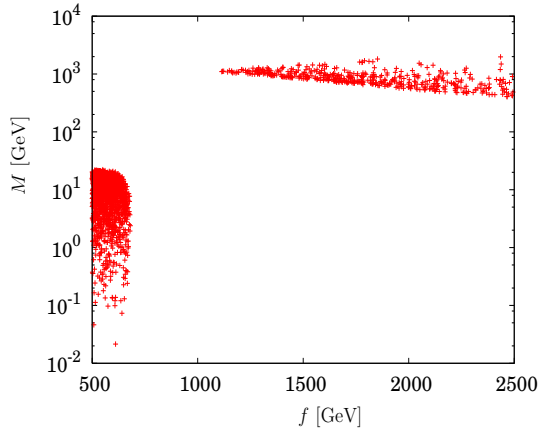
$$f, M, \mu_r, \hat{\mu}, \quad (18)$$

where  $M$  is the heavy top quark mass scale, both  $\mu_r$  and  $\hat{\mu}$  are soft symmetry breaking terms. The masses of the top and heavy top quarks are determined by  $f$  and  $M$  while those of the scalar particles  $\hat{h}_1$ ,  $\hat{h}_2$ ,  $\phi^\pm$  and  $\phi^0$  largely depend on  $\hat{\mu}$ ,  $\mu_r$  and  $f$ . Another scale  $\hat{f}$ , which is associated with the masses of the heavy gauge bosons, can be determined from the electroweak symmetry breaking condition: there is a generic relation between  $\hat{f}$  and  $f$  since Coleman-Weinberg potential of the Higgs boson mostly depends on  $M$ ,  $f$  and  $\hat{f}$ . For scalar potential, there is a tree level mass term proportional to  $\mu_r^2$ . So we may not acquire negative mass squared term which is necessary for electroweak symmetry breaking and it gives an upper bound for the value of  $\mu_r$ . For a given  $f$ ,  $\hat{f}$  becomes larger as  $M$  increases. It is because the increase of  $M$  contributes positively to the Higgs mass through the top loop while the increase of  $\hat{f}$  contributes negatively to the Higgs mass through the gauge boson loop, and thereby these contributions cancel out themselves in order to retain  $v = 246 \text{ GeV}$ . To draw a meaningful information on the model parameters from the  $\rho$ -parameter, we scan the parameter space generally, i.e.,

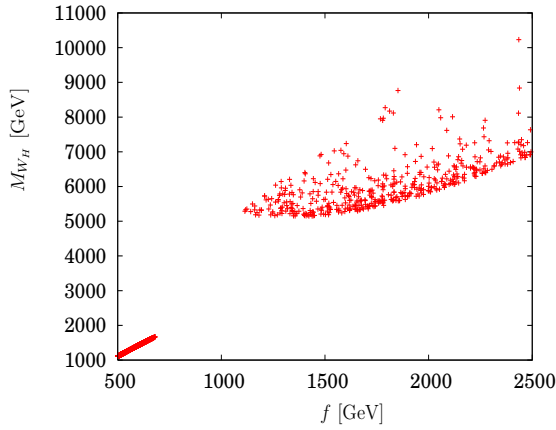
$$500 \text{ GeV} \leq f \leq 2500 \text{ GeV}, \quad 0 \leq M, \mu_r, \hat{\mu} \leq f. \quad (19)$$

Even though too large  $f$  makes the model unviable, we take the rather large value of  $f$ , 2.5 TeV, as an upper limit for completeness of the scanning. As a result of  $\rho$ -parameter calculation, we can obtain the allowed regions of parameter space. As an example, Fig. 1 shows the allowed regions of parameter space for  $f$  versus  $M$ . It is interesting to notice that the allowed parameter space is divided into two regions; less than 670 GeV and larger than 1100 GeV roughly, for  $f$ . This can be figured out as follows. The loop corrections tend to be larger as  $f$  increases. It is because the masses of the particles involved in one-loop correction increase in general as  $f$  increases. But at the same time, the mixing angles of top-heavy top quarks also vary. Since the mixing angles depend on not only  $f$  but also  $M$ , these two effects compete during the increase of  $f$ . Because of this interplay of top mixing angles and masses, we have two distinct allowed parameter spaces. For small  $f$ , solution points prefer very small values of  $M$ . It means there is no large mixing between the top and heavy top quarks. In general,  $\Pi^{WW}(0)$  is

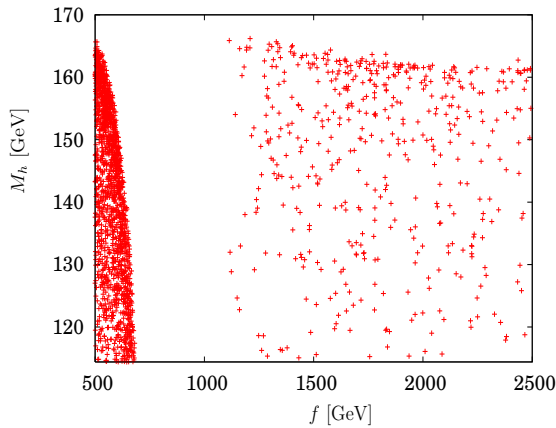
large for small  $f$ , and decreases as  $f$  increases. So for fitting the observed W-boson mass in the small  $f$  region, which is directly related to the  $\rho$ -parameter, we restrict the  $\Delta\hat{r}$  within rather small range. Because the  $\Delta\hat{r}$  is mostly determined by  $\Pi^{ZZ}(M_Z^2)$ , it should be also small. For doing that, we should take the small value of  $M$ , which makes the masses and mixing angles of heavy top quark small. We find that in the small  $f$  region,  $M$  should be smaller than about 22 GeV. Soft symmetry breaking parameter  $\mu_r$  is restricted to the values less than around 60 GeV. This bound arises mainly from the electroweak symmetry breaking condition, and is generically independent of the  $\rho$ -parameter. Another free parameters  $\hat{\mu}$  is not restricted from the one-loop corrected  $\rho$ -parameter. The reason is that  $\hat{\mu}$  only contributes to the masses of  $\hat{h}_1$  and  $\hat{h}_2$ , and their contributions are effectively cancelled among the relevant loop diagrams. This region of parameter space can provide constraints on the masses of many particles appear in this model. First, let us consider the masses of the heavy top and heavy gauge bosons. Their masses generically increase as  $f$  increases. The mass of the heavy top quark is uniquely determined when  $f$ ,  $\hat{f}$  and  $M$  are fixed. So does top Yukawa coupling. Basically  $\hat{f}$  is determined by the electroweak symmetry breaking condition, but their  $M$  and  $\mu_r$  dependence provoke the ambiguity on its value. For small  $f$  region, since  $M$  is also very small, the  $m_T$  is almost determined by  $f$  alone. For large  $f$  region, it becomes spread due to the top mixing angles. The plots of the heavy  $Z$  and  $W$  boson masses versus  $f$  are quite similar to that of the heavy top mass versus  $f$ . In the case of heavy  $W$  boson, the strongest constraint come from  $K_L - K_S$  mixing. The strongest bound ever known is  $m_{W_H} > 1.6 \text{ TeV}$ , with the assumption of  $g_L = g_R$  [16]. This can exclude some region from Fig. 2. In this case, small  $f$  region can be completely excluded. If the lower bound for  $f$  is confirmed, we can give the lower bound for  $f$  as 1.1 TeV from our calculation of the  $\rho$ -parameter and also for many particles appear in the model. Another constraints on the  $m_{W_H}$  from CDF and D0 are about  $650 \sim 786 \text{ GeV}$ , as lower bound [17, 18]. For Our results remain safe from these experimental bounds. Heavy  $Z$  boson has also been studied in detail by many experimentalists. Current experimental bound is about  $500 \sim 800 \text{ GeV}$  from precision measurements [13] and  $\sim 630 \text{ GeV}$  from CDF [13]. In this case, also safe is the mass of heavy  $Z$  boson. With the parameters allowed by the  $\rho$ -parameter, the masses of new scalar bosons  $\hat{h}_{1,2}$ ,  $\phi^0$  and  $\phi^\pm$  are also constrained.  $\hat{h}_{1,2}$  has almost degenerate masses, and are dependent on both  $\mu_r$  and  $\hat{\mu}$ , unlike the  $\phi^{0,\pm}$  which depend only on  $\mu_r$ . Their masses are seriously constrained according to the value of  $f$ . Unfortunately, we cannot give a lower bound on the mass of  $\phi^0$ . In fact, its mass, though it is quite small, arise from radiative corrections. For  $\phi^\pm$ , the loop contribution is rather large so it acquire larger mass compared to the neutral one. The  $\rho$ -parameter cannot give a strong restriction on the Higgs mass. In the whole space, Higgs mass is restricted below about 167 GeV. We cannot give a lower bound for Higgs bo-



**Fig. 1.**  $M$  vs.  $f$ , allowed range.



**Fig. 2.**  $m_{W_H}$  vs.  $f$ , allowed range.



**Fig. 3.**  $m_h$  vs.  $f$ , allowed range.

son mass from  $\rho$ -parameter itself. Here, we adopt the LEP bound for Higgs mass, 114.4 GeV [19], since its structure is same as the SM. The generic behavior of Higgs mass as a function of  $f$  is shown in Fig. 3.

## 4 Summary

We summarize the results of our analysis as follows. With the observed  $\rho$ -parameter, we see that the allowed parameter space is divided into two separate regions:  $f$  smaller than about 670 GeV and larger than about 1.1 TeV. We give the bounds on the mass spectrum of many particles for either region. Especially the heavy gauge bosons remain safe from the experimental constraints. Unlike the other particles, we cannot set a lower bound for the neutral  $\phi^0$  scalar. But loop correction plays an important role for the charged  $\phi^\pm$  scalars, yielding mass difference between the charged and neutral scalars. Further analysis is required in order to reduce the allowed region. If the small  $f$  region is excluded, for example by Ref. [10], we can provide exact lower bounds for the masses of  $T, Z_H, W_H, \hat{h}_{1,2}$ , and  $\phi^\pm$ . But even in that case, we cannot do so for  $\phi^0$  and SM Higgs boson.

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